Risk-Adjusted Profitability
Risk-Adjusted Profitability

Why?
- Decision making
  - Strategic planning – growth of business units
  - Bonus allotment
  - Risk management – hedging, reinsurance

General approaches
- Return on allocated capital
- Value-added
- Market value of risk

Will look at each – all have problems
Return on Allocated Capital

Capital allocation usually done by allocating risk measures

Requires:
- Selection of risk measures
- Allocation method

Problems
- Choice of risk measure not definitive
- Artificial to allocate – units not limited to allocated amount
- Not clear that return on allocated capital is financially accurate comparison of value of units
Value-Added

- Allocates capital costs to units and subtracts from profit

- For financial companies this is method of Merton-Perold – capital cost is an option price

- We call it capital consumption, after Astin paper by Don Mango

- Problems are in complexity of calculation

- If done right, comes down to allocation of firm value to business units
  - Requires valuation methodology
Market Value of Risk

- Sounds like right method in theory
  - Other methods can be considered approximations or approaches to calculating market value

- One problem is that value within a firm may be different than overall market value
  - Right measure may be effect of a business unit on market value of firm – same as for value-added

- Problem of requiring a valuation methodology

- All 3 methods end up at this issue
1. Capital Allocation
Capital Allocation Standards

- **Adds-up**
  - Sum of capital of units is capital of firm

- **Marginal effect**
  - Matches marginal cost of capital to marginal revenue, for proper decisions

- **Suitable (Tasche)**
  - Growing units with higher risk-adjusted return increases risk-adjusted return of firm

- **Value-based**
  - Allocated capital is proportional to value of risk of the unit
Capital Allocation Methodology

- Allocation methods
  - Co-measures
  - Marginal decomposition
  - Aumann-Shapley
  - Myers-Read

- Risk measures
  - Moment-based measures
  - Tail-based measures
  - Transformed probability measures
Allocation Methods for Risk Measures

Co-measures
- Additive decomposition into components

Marginal decomposition
- Subset of co-measures that for some risk measures in some conditions are marginal

Aumann-Shapley
- General method agreeing with marginal decomposition in some key cases

Myers-Read
- One popular method
Allocation by Co-measures

- **Goal is additive allocation**
  - Capital allocated separately to lines A and B will equal the capital allocated to lines A and B on a combined basis.

- Start with a risk measure for the company, for example the average loss in the 1 in 10 and worse years

- Then, consider only the cases where the company’s total losses exceed this threshold. In this example it is the worst 10% of possible results for the company.

- For these scenarios co-measure is how much each line of business is contributing to the poor results
Definition of Co-measures

Denoting loss for the total company as $Y$, and for each line of business as $X_i$ let:

- $\rho(Y) = \mathbb{E}[Y \mid F(Y) > \alpha]$ . Then co-TVaR is
- $R(X_i) = \mathbb{E}[X_i \mid F(Y) > \alpha]$

More generally:

- Risk measure $\rho(Y)$ defined as: $\mathbb{E}[h(Y)g(Y) \mid \text{condition on } Y]$, where $h$ is additive, i.e., $h(U+V) = h(U) + h(V)$

Allocate by $r(X_j) = \mathbb{E}[h(X_j)g(Y) \mid \text{condition on } Y]$

- VaR$_\alpha(Y) = \mathbb{E}[Y \mid F(Y) = \alpha]$, $r(X_j) = \mathbb{E}[X_j \mid F(Y) = \alpha]$
Example – Standard Deviation

- Not usually defined by expected value

- But take \( h(X) = X - EX \) and \( L(Y) = (Y - EY)/\text{Std}(Y) \). Then:

- \( \rho(Y) = E[(Y - EY)^2/\text{Std}(Y)] = \text{Std}(Y) \) and

- \( r(X_j) = \text{Cov}(X_j, Y)/\text{Std}(Y) \)

- Many risk measures can be put in this form
Marginal Decomposition of Risk Measures

- Marginal impact of a business unit on firm risk measure is decrease in overall risk measure from ceding a small increment of the line by a quota share.

- Marginal allocation assigns this marginal risk to every such increment in the line.
  - Treats every increment as the last one in.

- If sum of all such allocations over all lines is the overall firm risk measure, this is called a marginal decomposition of the risk measure.

- All co-measures are additive but not all are marginal.
Advantage 1 of Marginal Decomposition

You would like to have it so that:

- If you increase business in a unit that has above average return relative to risk
- Then the comparable return for the whole company goes up
- Tasche called this suitability

Not all allocation does that; marginal decomposition does

Thus useful for strategic planning
Advantage 2 of Marginal Decomposition

- Economic principle of comparing marginal price with marginal income
  - If you make more profit from selling a unit than it takes to make it, then keep making
    - Even though fixed costs maybe not covered

- Here profit from marginal increase in business is compared to marginal increase in firm risk that results
Formal Definition

Marginal \( r(X_j) = \lim_{\varepsilon \to 0} [\rho(Y+\varepsilon X_j) - \rho(Y)]/\varepsilon \).
- Take derivative of numerator and denominator wrt \( \varepsilon \).
- L’Hopital’s rule then gives \( r(X_j) = \rho'(Y+\varepsilon X_j) \rvert_0 \).

Consider \( \rho(Y) = \text{Std}(Y) \)
- \( \rho(Y+\varepsilon X_j) = [\text{Var}(Y)+2\varepsilon \text{Cov}(X_j,Y)+\varepsilon^2 \text{Var}(X_j)]^{1/2} \) so \( \rho'(Y+\varepsilon X_j) \rvert_0 = \)
- \([\text{Var}(Y)+2\varepsilon \text{Cov}(X_j,Y)+\varepsilon^2 \text{Var}(X_j)]^{-1/2} [\text{Cov}(X_j,Y) + \varepsilon \text{Var}(X_j)] \rvert_0 \)
- \( r(X_j) = \text{Cov}(X_j,Y)/\text{Std}(Y) \)

So co-measure gives marginal allocation

Not every co-measure does this
Example – Tail Value at Risk, etc.

- Co-TVaR, co-Var are marginal decompositions

- $\text{EPD}_\alpha = (1 - \alpha)[\text{TVaR}_\alpha - \text{VaR}_\alpha]$ is expected insolvency cost if capital = $\text{VaR}_\alpha$

- Co – EPD is $(1 - \alpha)[\text{co-TVaR} - \text{co-VaR}]$ and is marginal
Requirements for Marginal Decomposition

- Risk measure is homogeneous \( r_1 \) – or scalable: \( r(aX) = a r(X) \)

- Change in business is homogeneous
  - Reinsurer grows by taking higher shares of existing treaties
  - Small insurer quota-shares out net positions in every line and grows by reducing % ceded

This is rarely exactly true, but only approx.
Natural Co-measure Not Always the Marginal One

Let $\rho(Y) = E[Ye^{cY/EY}]$

Why divide by $EY$ in exponent?
- Makes it scalable
- But not a costless trick

Natural co-measure is $R(X) = E[Xe^{cY/EY}]$
- But this is not marginal
- Marginal co-measure is
  
  $r(X) = E[Xe^{cY/EY}] + c(EX/EY)E[Ye^{cY/EY}(X/EX– Y/EY)]$
Aumann-Shapley

- Game theory allocation
- Used in allocation of pooled costs in manufacturing
- Starts with \( \rho(tY) \) for \( 0 \leq t \leq 1 \) where every line \( X_j \) is scaled by same \( t \)
- Calculate marginal impact of \( tX_j \) on \( \rho(tY) \)
- Average these over all \( t \in [0,1] \)
- In homogeneous case this is marginal decomposition
- Even without homogeneity AS has some properties of marginal decomposition, but not suitability
- Not always marginal at current level (\( t=1 \))
Risk measure is capital

Constraint is in default every unit loses same % of expected loss

Finds the marginal allocation that maintains this constraint

I.e., if unit shrinks by an increment, capital is allowed to shrink to maintain same overall % of expected loss lost in default

That reduction in capital is allocation to unit

Adds up under homogeneity
Myers-Read Background

- Massachusetts government review of rates allows allocating frictional costs of holding capital
- Capital requirement viewed as producing target for % lost at default
- Myers-Read allocates frictional costs by impact on capital requirement
- But frictional costs not necessarily proportional to cost of bearing risk
  - Losses that do not cause default still can lose money and constitute risk to company
capital allocation to lines of business based on the Myers and Read approach is either not necessary for insurance rate making (in the case of no frictional costs) or even leads to incorrect loadings (when frictional costs are considered).

From the perspective of a regulatory authority the situation could be different. ... identical risks should have the same price that, additionally, guarantees an adequate safety level. ... both these goals are achieved by the Myers and Read approach.
Myers-Read Extension

- Powers: Using Aumann-Shapley Values to Allocate Insurance Risk: The Case of Inhomogeneous Losses, NAAJ coming

- Uses AS to carry out the Myers-Read scheme in the inhomogeneous case

- Different answer than Myers-Read

- Potential for application to other measures
Types of Risk Measures

- Moment based measures
  - Variance, std deviation, semi-std deviation
  - Generalized moments, like $E[Y e^{cY/EY}]$

- Tail based measures
  - Look only at the tail of the distribution

- Transformed probability measures
  - Risk measure functions of probability distributions
  - Change probabilities of results
  - Change probabilities of events
**Moment Based Risk Measures**

**Standard Deviation**
- If, for example, you are working with losses in Euro then standard deviation is a measure of the uncertainty also in Euro.
- Like variance it doesn’t distinguish between good and bad deviations

**Semi-Variance**
- $\text{Semi-Variance} = \mathbb{E}[(X - \mathbb{E}[X])^2 | X > \mathbb{E}[X]]$
- Measures only the uncertainty when losses are above average
- Gets more at real risk
- Square root is semi-standard deviation
Tail-Based Measures

- Probability of default
- Value of default put option
- Value at risk
- Tail value at risk
- Excess tail value at risk
- Weighted excess tail value at risk
Probability of Default

- A long-standing actuarial concept
- But it is beyond the ability of current models to quantify
  - Role of underwriting practices, fraud, mismanagement big in insolvency but hard to measure
  - Loss models themselves not that accurate way out in tail
- Default put value is market value of the losses beyond default
  - Similar calculation problems as default probability
- Impairment probabilities more practical
  - How much of surplus is lost in 1-in-10, 1-in-100, etc.
  - Probability of drop in surplus and average drop when there is one
  - Capital can be set as multiple of losses at various impairment levels
Value at Risk

- Marketing name for a percentile of the loss distribution
- Single percentile a very limited look at risk
- Arbitrary – no particular probability stands out
- Hard to analyze into components
  - In a simulation, nearby losses could have very different causes and line breakouts
- Mistakenly thought to represent loss by return period
  - But if 90th percentile loss happened every 10 years, you would never have the 99th percentile loss
Tail Value at Risk = Conditional Tail Expectation

- Average loss at target probability and beyond
- This one does represent the loss at a return period
- More stable breakout into components as not too sensitive to single loss scenarios
- Still arbitrary choice of probabilities
  - Economically meaningful choices are probability of default and probability of any surplus loss
  - Latter is perhaps best – possible to measure and includes all larger loss scenarios
  - 99% used a lot but arbitrary and probably too far out
- Problem of linear treatment of all larger losses – contrary to usual ideas of risk preferences
  - Alternative is to take expectation using transformed probabilities – may represent economic value of tail losses

Excess TVaR is excess of TVaR over mean
When to use TVaR versus XTVaR?

- **XTVaR** is used when using incurred losses only
  - Measures the extent loss exceed expectations (or plan)
    - Capital is needed to cover the losses above average
    - A reduction in capital typically happens when losses are at the 80% or higher then
      - 4 out of 5 years a company is profitable, 1 out of 5 years the company loses money

- **TVaR** is used when using underwriting results (U/W)
  - Measures the amount of underwriting loss
    - Ignoring investment income, an U/W loss will result in a reduction in capital
    - Or could be done on net profit/loss in total
EPD – Expected Policyholder Deficit

- Can be defined at any tail probability $\alpha$, like 10%, 1%, etc.
- Can be calculated as $\alpha[\text{TVaR}_\alpha - \text{VaR}_\alpha]$
- Represents expected loss beyond VaR
- Unconditional tail, whereas VaR and TVaR are conditional
- If $\alpha$ is probability of default, this is expected value of policyholder shortfall
  - If transformed probability distribution is used this could be the value of the default put option
Transformed Probability Measures

- Spectral measures are functions of probability distributions
  \[ \rho = E[Y \cdot \eta(F(Y))] \] for nonnegative function \( \eta \).

- Distortion measures change probabilities of results, using \( S(x) = 1 - F(x) \)
  \[ \rho(X) = \int_0^\infty g[S(x)]dx \] for \( g(p) \) a cdf on \([0,1]\)

- Change probabilities of events
  Underlying frequency and severity probabilities shifted towards more losses then mean or other risk measure calculated
Spectral Measures

- $\rho = E[Y \cdot \eta(F(Y))]$ for nonnegative function $\eta$.
- $\eta(p) = \begin{cases} 0, & p \leq q \\ 1/(1-q), & q < p \end{cases}$ gives TVaR$_q$

- TVaR$_q = E[Y \mid F(Y)>q] = \int_{y > F^{-1}(q)} y f(y)/(1-q)dy$

- $\eta(p) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{p-(1-q)}{\sigma}\right)^2\right)$ gives blurred VaR

- Can blur VaR with a uniform as well
Distortion Measures

Distortion measures change probabilities of results, using \( S(x) = 1 - F(x) \)

\[ \rho(X) = \int_0^\infty g[S(x)]dx \] for \( g(p) \) a cdf on [0,1]

\( g[S(x)] = S^*(x) \) is a survival function so \( \rho(X) \) is mean with pdf \( f^* = g'[S(x)]f(x) \)

Thus \( \int_0^\infty y f^*(y)dy = \int_0^\infty yg'[S(y)]f(y)dy = E[Y \eta(F(y))] \) with \( \eta(p) = g'(1-p) \), and so \( \rho \) is a spectral measure

If \( g(p) \geq p \) then \( \rho(X) = \int_0^\infty g[S(x)]dx \geq \int_0^\infty S(x)dx = E(X) \) so \( \rho(X) \) is a loaded mean
Distortion Measures

- $g(p) = p^a$, $0 < a < 1$, is the proportional hazards, or PH, transform, so called because it changes $\log S(x)$ by a factor

- Wang transform with parameters $a$, $b$:
  
  $g(p) = 1 - T_a[\Phi^{-1}(1-p) - b]$

  $T_a$ is the t-distribution function with $a$ degrees of freedom, $a$ not necessarily an integer, with $\Phi$ standard normal distribution

  $b \sim 0.45$ and $a \sim 5.5$ has fit bond pricing
Change Event Probabilities

- Required for arbitrage-free prices
  - Distortion measures are subadditive and co-monotonically additive but not additive
  - Some think this is better than arbitrage-free in that it reflects risk reduction from pooling
  - But market prices should already reflect pooling and charging more than market is unlikely

- Prices for risky instruments in practice and theory have been found to be approximated by changing event probabilities
  - Esscher transform for compound Poisson process tested for catastrophe reinsurance
  - Black-Scholes and CAPM can be expressed as transforms

- Transformed probability measures have potential for being proportional to the market value of the risk
Possible Transforms

- **Compound Poisson martingale transform**
  - Requires function $\phi(x)$, with $\phi(x) > -1$ for $x > 0$
  - $\lambda^* = \lambda[1 + E\phi(X)]$
  - $g^*(x) = g(x)[1 + \phi(x)]/[1 + E\phi(X)]$

- **Entropy Transform = Esscher Transform for Compound Poisson**
  - $g^*(y) = g(y)e^{cy/EY}/Ee^{cY/EY}$
  - $\lambda^* = \lambda Ee^{cY/EY}$
Entropy Transform = Esscher Transform for Compound Poisson
Comparison to reinsurance prices

Quadratic

Average

Expected Loss on Line

Loading

 MMM Loading

 MEM Loading

 Mixed Loading

 Premium Loading

Loading Factors for Martingale Pricing of FE

Expected Loss on Line

Guy Carpenter
Which Risk Measures?

- Homogeneous good for allocation
  - Almost all of above are

- Useful to be proportional to value of risk being measured
  - Favors transformed probability measures

- Tail measures are popular but ignore some of the risk
  - A risk worth charging for is a risk worth measuring

- Transforming event probabilities marginal even in non-homogeneous growth case
Comment on Allocation
Grundl & Schmeiser

we could not find reasons for allocating equity capital back to lines of business for the purpose of pricing.

every capital allocation method that distributes the cost of equity capital to the different lines in the given structure of the company is an arbitrary way of common cost allocation. The allocation of common costs... typically leads to wrong decisions by an insurance company.
My Position

- For companies that want to allocate capital, use marginal decomposition, preferably with a risk measure based on transformed probabilities of underlying events.

- Like dentists who recommend sugarless gum to their patients who chew gum.
2. Value-Added
Alternative to Capital Allocation (for measuring risk-adjusted profit)

- Charge each business unit for its right to access the capital of the company (consuming capital)
  - Profit should exceed value of this right
  - Essentially a value added approach
  - Avoids arbitrary, artificial notions of allocating capital

- Business unit has option to use capital when premiums plus investment income on premiums run out (company provides stop-loss reinsurance at break-even)
  - Company has option on profits of unit if there are any
  - Pricing of these options can determine value added
  - Combination of both is not a contingent claim
Some Approaches to Valuing

- Not a simple option – no fixed date or amount
- Units that have big loss when overall firm does cost more to reinsure, so correlation is an issue
- Bounds on worth of stop loss
  - Probably worth more than expected value
  - Probably worth less than market value
    - Stop-loss pricing includes moral hazard
    - Company should be able to control this for unit
- Or look at impact of unit loss on firm value
  - Ideal information, if you can get it
Impact on Firm Value Example

g(x) is change in value due to change in capital
Falls off sharply for large losses
Hypothetical curve formula not shown but used in examples
### Value Added – Risky Company Gross

#### 4 Possible Profit Scenarios for 3 Lines

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<th>Gross Economic Profit</th>
<th>Scenario:</th>
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<th>3</th>
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#### Capital Charge

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#### Contribution

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Expected profit but risk adjusted impact is negative.

Expected value change is negative due to big drop from the very large loss scenario.
Same after Reinsurance

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Gave up 20% of mean profit (40% for Homeowners) but now expected value change is positive.

Also each line is making a positive contribution.
How to do it in practice?

- We don’t really know $g(x)$ function
- Best bet now is probably pricing of the implicit stop loss
- Could do that with any pricing methodology once the losses are modeled
  - Expected losses + 30% of standard deviation
  - Expected losses under minimum entropy measure
- Profit is an option too but $(\text{profit} - \text{cost of capital})$ is not
  - Valuing that is value-added of business unit
  - Sum is value of firm, so really allocating firm value
Value Added Summary

- Perhaps more theoretically sound than allocating capital
- Does not provide return on capital by unit
- Instead shows value of unit profits after accounting for risk
- A few approaches for calculation possible
- Comes down to calculating firm value and allocating that to business unit
3. Market Value of Risk
Market Value of Risk Transfer

- Needed for right risk measure for capital allocation
- Needed to value options for capital consumption
- If known, could compare directly to profits, so neither of other approaches would be needed
Two Paradigms

- CAPM
- Arbitrage-free pricing

And their friends
Insurance risk is zero beta so should get risk-free rate?

But insurance companies lose money on premiums but make it up with investment income on float
   ✦ Really leveraged investment trust, high beta?

Hard to quantify
   ✤ Cummins-Phillips using full information betas found required returns around 20%
Problems with CAPM (besides estimation)

- How to interpret Fama-French?
  - Large cap vs. small cap alternate in favor
  - Proxies for higher co-moments?
  - Could co-moment generating function work?

- What about pricing of jump risk?
  - Earthquakes, hurricanes, ...
  - Two standard approaches to jump risk:
    - Assume it is priced
    - Assume it is not priced
  - Possible compromise: price co-jump risk
Arbitrage-Free Pricing

- Incomplete market so which transform?
- Same transform for all business units?

Related methods

- Distortion measures – not arbitrage-free but still use probability transforms
  - Weaker assumption than arbitrage-free
- No good deals
  - Stronger assumption than arbitrage-free
No Good Deals

- Rules out arbitrage and good deals

- Good deals have some risk but so much more potential reward that anyone would take the deal

- What is a good deal is defined by some arbitrary standard – maybe 7 flavors already – but gives more restricted pricing ranges than no arbitrage
So in Conclusion …

- Marginal decomposition with co-measures improves allocation exercise
- Choice of risk measure can make result more meaningful
- Capital consumption removes some arbitrary choices and artificial notions of allocation
- Market value of risk is what is needed in each method – but we don’t really know how